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Topological Complexity Measures of Supply Chain Networks

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Abstract

In this article, we firstly present an architectural framework for supply chain networks. Then, convergent supply chain models that are typical for assembly processes are divided into classes on the basis of the numbers of initial components. Subsequently, selected indicators for measuring topological complexity of assembly process are employed. The indicators used are benchmarked based on computational experiments. Finally, pertinent findings from this exploration are commented and some related future research directions are outlined.

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Keywords: Assembly system; Structural complexity; Vertex degree

1. Introduction

Global supply chain networks are continually developing and expanding nationally. At the same time, supply chain networks are becoming more complex as manufacturing companies expand their economic cooperation. Evenly, assembly supply chains (ASCs) are becoming increasingly complex not only due to technological advancements but also as a result of the changes made to attract more customer interest. Therefore, assembly processes of OEMs must be optimized accordingly in order to increase the performance of the manufacturing system while respecting sustainable manufacturing (SM) principles. SM brings benefits and cost but can make companies more competitive, and in turn bring more resources into the business.

In this paper we consider the problem of topological complexity measurement of convergent assembly processes to find the least complex and sustainable designs in manufacturing area. As first step, we have described a methodological framework for the design of the assembly models based on generation of rooted trees. These supply chain models are divided into classes on the basis of the numbers of initial components. Subsequently, selected two indicators for measuring topological complexity of assembly process are proposed and analyzed. The indicators used are compared and analyzed based on computational experiments.

Finally, relevant findings from this exploration are formulated.

2. Related work

Assembly supply chains can be defined as interconnections of workstations that relate to each other through upstream and downstream linkages, in the different processes and activities that produce value in the form of products in the hands of the ultimate consumer [1]. Competition at company level has been replaced with competition of concurrent supply chains [2]. Performance models and their improvement have to take all chain aspects and processes into account. Moreover, it can be stated that the level of the network consists of facilities and different types of flows (material, information flows, etc.) and contribute to the overall complexity of the chain. The system complexity can be viewed from many aspects, some of which are directly or indirectly linked. In terms of sustainable manufacture it is important to follow and develop designs with minimal negative environmental and economic impact. Therefore, challenges in finding the best sustainable supply chain design is currently subject or researches. The first effective complexity measures have been developed some

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time ago. For example, algorithmic information complexity derived by Kolmogorov [3] and Chaitin [4] is linked with Shannon information entropy [5]. Both theories use the same unit (bit) as a measure of information to describe any system. The more facilities and their uncertain behaviour there are in the system, the more information is needed to describe the structure/system and the bigger is the entropy. Another category is the stochastic complexity applying the concept of minimum description length principle, e.g. [6]. These theories define system complexity as the minimum information description size in bits.

According to Strogatz [7], structural properties of the complex networks are the most basic issues since they always affect the function. Moreover, he added that there are missing unified approaches to underlay and uncover the topology of such networks related to complexity.

Assembly supply chain design and management can be very difficult since different sources of uncertainty are combined with combinatorial and topological aspects of ASCs. Such uncertainty may arise as a result of customer specification and its variability resulting in unreliability at external suppliers, which is normal at customized productions. In this context, various deterministic and stochastic topological models have been developed so far by different authors [8-9] but with no relevant implication on SC control and management while taking the system complexity into account. Possible way to cope with challenges and specific customer needs is through development of effective supply chain performance models and measures to provide them with product in shortest period of time. In order to have that, a mutual comparison of selected structural and axiomatic design-based (AD) measures was provided and graphical and numerical correlations have been obtained in order to decide about the most suitable topological complexity measure of ASCs.

3. Generating all possible assembly structures

Global supply chain networks are continually developing and expanding nationally. Under convergent assembly structure we understand that one in which each node in the chain has at most one successor, but may have any number of predecessors. Such assembly structures can be divided into two types, modular and non-modular. In the modular structure, the intermediate sub-assemblers are understood as assembly modules, while the non-modular structure consists only from suppliers (initial nodes) and a final assembler (end node). The framework for generating topological classes of assembly structures follows the work of Hu et al. [10], who outlined the way forward to model possible supply chain structures with four original suppliers in terms of production variety and in relation to product. Generation of all possible combinations of arbitrary class or rooted trees brings enormous combinatorial difficulties.

Thus, it is also assumed that each such assembly graph satisfies the following conditions. Let t be a node of rooted tree T and there is only one path from t to root r_t. The root has degree ≥ 2. Further we assume that vertex t of T with degree 1 is initial node of a path, and we denote number of initial nodes by i, where $i \geq 3$. Trees with the same number of initial nodes will be grouped in a corresponding class C_i. Finally, it is supposed that each ancestor node of T has degree ≥ 3.

Classes of the rooted trees begin with a class C_3, because the class C_2 is represented by only single graph. The class C_3 represents rooted trees that have initial nodes $i = 3$. In this simple case it is clear that the number of the trees equals two graphs since the number of partitions $p(3) = 3$ and its composition $p_3(3)$ does not meet specified conditions, because the root in given tree has degree 1. Then the number of the trees for the given class follows the expression $p(3) - p_3(3) = 3-1 = 2$. Based on this we can generate of all possible rooted trees (see example in Fig. 1).

Analogically, we can use pertinent partitions of $p(4) - p_4(4) = 5-1 = 4$ to determine an initial set of rooted trees for subsequent class C_4 (see Fig. 2, in the frame).

This class of trees consists also from composition (3, 1) represented by graph No.4. Its part $\lambda=3$ can be partitioned into two partitions (3) and (2, 1) and expressed by formula $p(3) - p_3(3)$. The partition $p_3(3)$ is already represented by graph No. 4 in Fig. 2 and the partition $p_2(3)$ has to be represented by additional graph No.5 in Fig. 2, which is missing to complete all possible graphs in the class C_4. Then a sum of all possible graphs in this class is 5.

Based on above description we can generate so called initial partitions and related graphs for arbitrary class C_i through partitions expressed by formula $p(n)-p_n(n)$. Subsequently, in order to obtain all possible rooted trees for arbitrary class it is necessary to multiple each partition in which at least one part $\lambda \geq 3$ by specific multiplication number as it was shown in case of class C_4.

Then, all possible assembly structures for given number of initial nodes can be created. An example of all possible rooted trees for the classes from C_2 to C_5 is shown in Fig. 3. The rooted tree partition presented in this section is further developed and applied on so called Vertex degree partition developed by authors in Section 4.

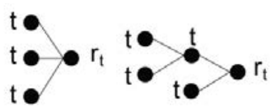


Fig. 1. All possible rooted trees for the class C_3.

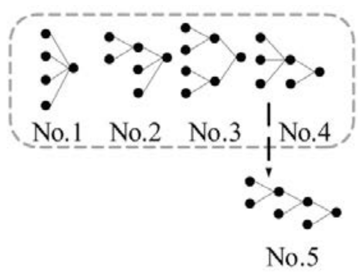


Fig. 2. Initial set and additional graph of rooted trees for the class C_4.

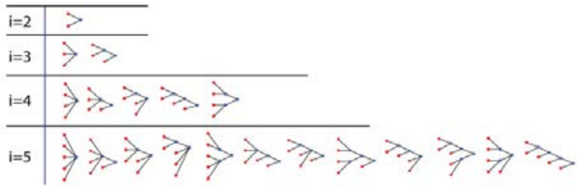


Fig. 3. All possible rooted trees of the selected classes from C_2 to C_5.

4. Application of Vertex degree index for structural complexity measurement

4.1. Theoretical background

According to Shannon's information theory [5], *the entropy of information H(α)* in describing a message of N system elements (or symbols), distributed according to some equivalence criterion α into k groups of N_1, N_2,…, N_k elements, is calculated by the formula:

$$H(\alpha) = -\sum_{i=1}^{k} p_i log_2 p_i = -\sum_{i=1}^{k} \frac{N_i}{N} log_2 \frac{N_i}{N}, \qquad (1)$$

where p_i specifies the probability of occurrence of the elements of the i^{th} group. Since it is of interest to characterize *entropy of information* of a network according to equation 1, it is possible to substitute symbols or system elements for the vertices. In order to define the probability for a randomly chosen system element i it is possible to formulate general weight function as $p_i = w_i / \Sigma w_i$, assuming that $\Sigma p_i = 1$.

Author [11] claims that, considering the system elements, the vertices and supposing the weights assigned to each vertex to be the corresponding vertex degrees, one easily distinguishes the null complexity of the totally disconnected graph from the high complexity of the complete graph. Then, the probability for a randomly chosen vertex i in the complete graph of V vertices to have a certain degree $\deg(v)_i$ can be expressed by formula:

$$p_i = \frac{\deg(v)_i}{\sum_{i=1}^{V} \deg(v)_i}, \qquad (2)$$

Based on our previous experiences the most feasible indicator to configuration complexity of general structures seems to be *Vertex Degree Index* (I_{vd}). The information entropy of a graph with a total weight W and vertex weights w_i can be expressed in the form of the equation:

$$H(W) = W\, log_2 W - \sum_{i=1}^{V} w_i\, log_2 w_i, \qquad (3)$$

Since the maximum entropy is when all w_i=1, then:

$$H_{max} = W\, log_2 W, \qquad (4)$$

By substituting $W=\sum \deg(v)_i$ and $w_i = \deg(v)_i$, the information content of the vertex degree distribution of a network called as Vertex degree index (I_{vd}) is derived by [11] that is expressed as follows:

$$I_{vd} = \sum_{i=1}^{V} \deg(v)_i log_2 \deg(v)_i, \qquad (5)$$

4.2. Complexity measurement based on I_{vd}

For the purpose of structural complexity measurement it is useful for all possible assembly structures to assign the corresponding numerical combination by vertex degree parameter, as shown in Fig. 4. Subsequently, for each of class based on numerical combinations, the number of non-repeated ASC structures can be obtained. These numbers follow the integer sequence A139582 by Omar [12].

Based on the I_{vd} parameter corresponding fragment of table with classes C_2 to C_7, non-repeated vertex degrees and their values of I_{vd} complexity can be seen in Fig. 5.

5. Application of Vertex degree index for structural complexity measurement

In order to apply the new AD based measure for quantification of topological complexity of ASCs, theoretical preconditions of design solutions and transformation related design matrices have to be explained and defined.

5.1. Transforming ASC structures into AD matrices

The main definition of Axiomatic design [13] states that any process can be seen in four main domains: process, functional, customer and physical. The process consists of several steps and at the end results with structured relations between customer needs, functional requirement (FR) and selected design parameters (DP). These relations or dependencies between FRs and DPs within any design hierarchy can be expressed by the relation:

$$FR = [A]\, DP, \qquad (6)$$

where each element of the matrix [A] can be expressed as

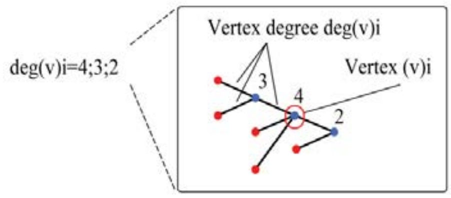


Fig. 4. Determination of vertex degree parameter by corresponding numerical combination.

i=2		i=3		i=4		i=5		i=6		i=7	
deg (v)$_i$	I_{vd}	deg (v)$_i$	I_{vd}	deg (v)$_i$	I_{vd}	deg (v)$_i$	I_{vd}	deg (v)$_i$	I_{vd}	deg (v)$_i$	I_{vd}
2	2	3	4,75	4	8,00	5	11,61	6	15,51	7	19,65
1		3;2	6,75	4;2	10,00	5;2	13,61	6; 2	17,51	7;2	21,65
		2		3;3	9,51	4;3	12,75	5;3	16,36	6;3	20,26
				3;3;2	11,51	4;3;2	14, 75	5;3;2	18,36	6;3; 2	22,26
				4		3;3;3	14,26	4;4	16,00	5;4	19,61
						3;3;3;2	16,26	4;4;2	18,00	5;4;2	21,61
						6		4;3;3	17,51	5;3;3	21,12
								4;3;3;2	19,51	5;3;3; 2	23,12
								3;3;3;3	19,02	4;4; 3	20,75
								3;3;3;3;2	21,02	4;4; 3;2	22,75
								10		4;3;3;3	22,26
										4;3;3;3;2	24,26
										3;3;3;3;3	23,77
										3;3;3;3;3;2	25,77
										14	

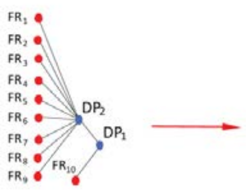
Fig. 5. Non-repeated sets of ASC structures based on vertex degree parameter.

A=FR/DP. Equation 6 can be expressed as each FR on the product component depends on the specific design parameter (DP) of the product specified by customer, so that each such dependency [*A*] can be understood as existing relation of FR on DP. If in the design matrix of any process element A refers to “0”, then FR is not in relation with DP. And vice versa for “1”, where there is relation between the DP and FR.

According to this approach, we indicate each initial node of the ASC model as FR (for example FR_1 to FR_{10} at C_{10}) and each sub-assembly vertex as DP (for example DP_1 to DP_3 depending on the specific ASC structure at C_{10} shown in Fig. 6a and 6b). This is because initial nodes practically represent company requirements on suppliers and specify the number of initial nodes into ASC. Subsequently, DPs are determined by these FRs as sub-structures. This way, the transformation of all repeated ASC is possible and valuable.

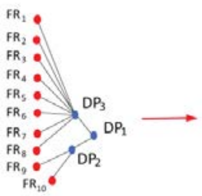
Analogically, we can transform each ASC structure into axiomatic design matrix (see examples in Fig. 6a for 10 FRs and 2 DPs and Fig. 6b for 10 FRs and 3 DPs).

Presented design matrices have been transformed as coupled designs. For such matrices it is characteristic that individual elements *[A]* are mostly non-zero and thus the FRs cannot be satisfied independently.



	DP_1	DP_2
FR_1	1	1
FR_2	1	1
FR_3	1	1
FR_4	1	1
FR_5	1	1
FR_6	1	1
FR_7	1	1
FR_8	1	1
FR_9	1	1
FR_{10}	1	0

a)



	DP_1	DP_2	DP_3
FR_1	1	0	1
FR_2	1	0	1
FR_3	1	0	1
FR_4	1	0	1
FR_5	1	0	1
FR_6	1	0	1
FR_7	1	0	1
FR_8	1	0	1
FR_9	1	1	0
FR_{10}	1	1	0

b)

Fig. 6. (a) ASC structure with 10 FRs and 2 DPs, (b) ASC structure with 10 FRs and 3 DPs, both transformed into design matrix.

5.2. Quantifying structural complexity of ASC applying entropy and disorder concepts

Different authors, e.g. [14-15], have discussed relations between entropy, disorder, chaos and even complexity. According to Martin et al. [16], the concepts of entropy and disorder are inherently linked, but disorder is only a metaphor for entropy but not the definition. On the other hand, chaos can be more or less strictly defined as it deals with deterministic systems whose trajectories diverge over time. This property can be found also in complex systems whose properties are still an important area of research.

Guenov [17] introduced three complexity indicators for architectural design. These indicators are relatively simple and easy to apply, and are sufficiently accurate for the early stages of systems design or redesign.

The first measure, introduced by Guenov [17] is denoted by authors as Systems Design Complexity (*SDC*), can be expressed as follows:

$$SDC = \sum N_j \ ln \ N_j, \qquad (7)$$

where the volume is equal to unity and N_j is interpreted as number of interactions per single DP of measured designed matrix.

The second measure is omega ‘Ω’ derived out of the entropy theory, and is called a Degree of disorder. It is expressed as follows:

$$\Omega = C_N^{N_1} * C_{N-N_1}^{N_2} * C_{N-N_1-N_2}^{N_3} \dots * \dots C_{N-N_1-\dots-N_{K-1}}^{N_K} = \frac{N!}{N_1! N_2! \dots N_K!}, \qquad (8)$$

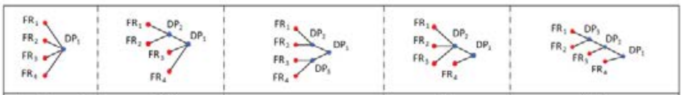
where ‘N’ is number of interactions within whole transformed design matrix, and N_1, N_2, …, N_K are number of interactions or “1s” per each DP of the same matrix.

Subsequent indicator $Ln\Omega$ has been proposed in order to obtain less exponential values of Ω.

Our main research question was: “How these topological complexity indicators could be effectively used to assess structural complexity of ASC?”

In order to find answer to the question we firstly apply the three presented measures and compare them with above mentioned I_{vd} index. For that purpose the above 5 non-repeated ASC structures of C_4 have been used to calculate complexity measures (see Tab. 1 above).

Tab. 1. Transformation of non-repeated ASC structure of C_4 with appropriate values of complexity indicators (I_{vd} and $ln\Omega$).



deg(v)	4	3;3	3;3;2	4;2	3;3;2
Ivd	8	9,51	11,51	10	11,51
lnΩ	0	2,71	6,04	3,56	7,14
SDC	5,5	6,93	8,32	8,84	10,23

	DP_1
FR_1	1
FR_2	1
FR_3	1
FR_4	1

	DP_1	DP_2
FR_1	1	1
FR_2	1	1
FR_3	1	0
FR_4	1	0

	DP_1	DP_2	DP_3
FR_1	1	1	0
FR_2	1	1	0
FR_3	1	0	1
FR_4	1	0	1

	DP_1	DP_2
FR_1	1	1
FR_2	1	1
FR_3	1	1
FR_4	1	0

	DP_1	DP_2	DP_3
FR_1	1	1	1
FR_2	1	1	1
FR_3	1	1	0
FR_4	1	0	0

As can be seen from Tab. 1, DPs are represented by number of ancestor nodes and the root (columns *N*), while all product components in the form of initial nodes are represented by FRs. Indicator I_{vd}, presented earlier [18-20], is considered by authors as the most suitable topological measure of ASCs. Other two topological measures $Ln\Omega$ and *SDC* revealed important differences in complexity of the networks represented by structures No. 3 and No. 5 in Table 1 with vertex degree $deg(v)_i$=3;3;2 while value of I_{vd} remains the same.

Subsequently, values of $Ln\Omega$ and *SDC* for classes C_2 to C_6 have been determined as can be seen in Tab. 2.

6. Comparison of complexity measures

Based on above-obtained sequence, it is possible to compare structural complexity between concurrent AS structures of the same class in order to have chance to be selected by given criterion. Alternatively, we can identify differences of structural complexity among arbitrary AS structures which belong to different classes.

Graphical comparison of selected two benchmarked indicators (I_{vd} and *SDC*) is shown in Fig. 7. Obtained results are described in the next section.

7. Conclusions

From tested newly applied indicators ($Ln\,\Omega$ and *SDC*) especially SDC indicator was recognized as useful measure. The reason why we identified its purposefulness can be extracted from the following findings:

1. According to SDC, non-modular structure in each class reaches the lowest complexity while according to I_{vd}, this logical assumption is not confirmed.

Tab. 2. Fragment of I_{vd}, $Ln\Omega$, *SDC* complexity values of transformed non-repeated structures for classes C_2-C_6 sorted according to $deg(v)_i$.

Non-repeated ASC graphs for C_2 to C_6					
Class	No.	$deg(v)_i$	I_{vd}	lnΩ	SDC
C_2	1	2	2	0	1,39
C_3	1	3	4,75	0	3,30
	2	3;2	6,75	2,30	4,68
C_4	1	4	8	0	5,55
	2	4;2	10	3,56	8,84
	3	3;3	9,51	2,71	6,93
	4	3;3;2	11,51	7,14	10,23
C_5	1	5	11,61	0	8,05
	2	5;2	13,61	4,84	13,59
	3	4;3	12,75	4,03	11,34
	4	4;3;2	14,75	10,23	16,89
	5	3;3;3	14,26	7,83	12,73
	6	3;3;3;2	16,26	14,74	18,27
C_6	1	6	15,51	0	10,75
	2	6;2	17,51	6,14	18,80
	3	5;3	16,36	5,35	16,30
	4	5;3;2	18,36	13,35	24,34
	5	4;4	16	4,43	14,05
	6	4;4;2	18	12,03	22,09
	7	4;3;3	17,51	11	19,59
	8	4;3;3;2	19,51	20,06	27,64
	9	3;3;3;3	19,02	15,66	20,98
	10	3;3;3;3;2	21,02	25,31	29,03

2. According to SDC it is possible in each class of ASC to identify higher number of unique complexity values unlike with I_{vd}.

3. Disadvantage of indicator $Ln\Omega$ is that for each non-modular structure, complexity equals zero.

4. Another disadvantage of indicator $Ln\Omega$ is that it allows us to identify lesser number of unique structural complexities than according to *SDC*.

Described statements can be easily verified and the

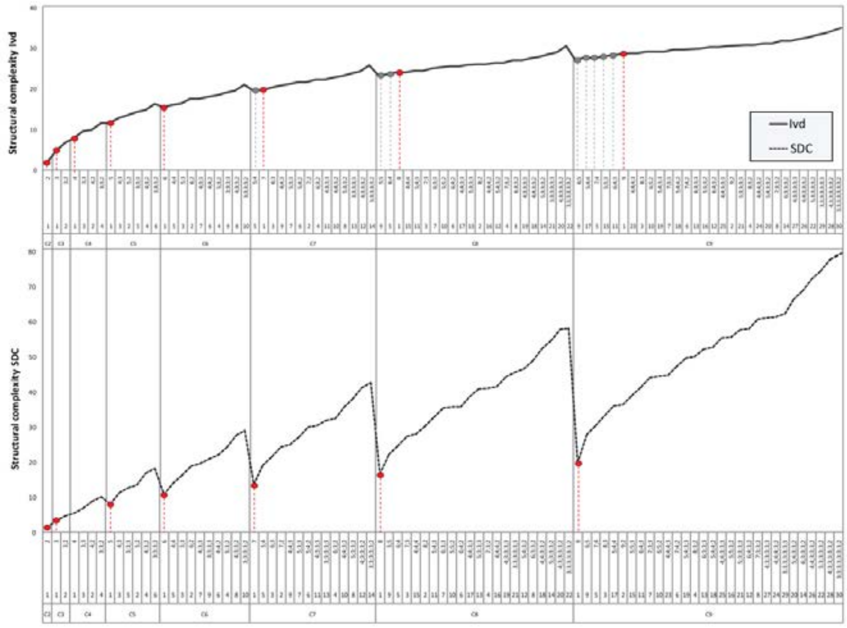


Fig. 7. (a) I_{vd} and *SDC* complexity curves of transformed non-repeated structures of classes C_2-C_9 sorted according to I_{vd} and individual classes, (b) comparison of I_{vd} and *SDC* complexity curves of transformed non-repeated structures of classes C_2-C_9.

properties of *SDC* indicator bring new perspectives in effort to develop effective tools for complexity management. Evidently, it will be useful to study other properties of relevant topological complexity indicators that can identify new pertinent findings.

Acknowledgements

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